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## **Energy Shaping Revisited**

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*Published in:*

Proceedings of the IEEE International Conference on Control Applications, 2000

**IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.**

*Document Version*

Publisher's PDF, also known as Version of record

*Publication date:*

2000

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Ortega, R., Schaft, A. J. V. D., Mareels, I., & Maschke, B. (2000). Energy Shaping Revisited. In *Proceedings of the IEEE International Conference on Control Applications, 2000* (pp. 121-126). University of Groningen, Johann Bernoulli Institute for Mathematics and Computer Science.

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# Energy Shaping Revisited

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## Abstract

Energy shaping is the essence of passivity based control (PBC), a controller design technique that is very well-known in mechanical systems. Our objectives in this article are threefold: First, to call attention to the fact that PBC does not rely on some particular structural properties of mechanical systems, but hinges on the more fundamental (and universal) property of energy balancing. Second, to identify the physical obstacles that hamper the use of “standard” PBC in applications other than mechanical systems. In particular, we will show that “standard” PBC is stymied by the presence of unbounded energy dissipation, hence it is applicable only to systems that are stabilizable with passive controllers. Third, to revisit a PBC theory that has been recently developed to overcome the dissipation obstacle, as well as to make the incorporation of process prior knowledge more systematic. These two important features allow us to design energy based controllers for a wide range of physical systems.

## 1 Introduction

The purpose of this article is to call the attention to the importance of incorporating energy principles in control. To achieve our objective, we propose to abandon the signal processing perspective of control and instead adopt the behavioral framework proposed by Willems [1]. In Willems’s far-reaching interpretation of control, we start from a mathematical model obtained from first principles,

say, a set of higher order differential equations and some algebraic equations. Among the vector of time trajectories satisfying these equations are components that are available for interconnection. The controller design then reduces to defining an additional set of equations for these interconnection variables to impose a desired behavior on the controlled system. We are interested here in the incorporation into this paradigm of the essential energy component. Therefore, we view dynamical systems (plant and controller) as *energy-transformation* devices, which we interconnect (in a power-preserving manner) to achieve the desired behavior.

## 2 Passivity and Energy Shaping

We are interested here in lumped-parameter systems interconnected to the external environment through some *port power variables*  $u \in \mathcal{R}^m$  and  $y \in \mathcal{R}^m$ , which are conjugated in the sense that their product has units of power (e.g., currents and voltages in electrical circuits, or forces and velocities in mechanical systems). We assume the system satisfies the *energy-balance equation*

$$\underbrace{H[x(t)] - H[x(0)]}_{\text{stored energy}} = \underbrace{\int_0^t u^\top(s)y(s)ds}_{\text{supplied}} - \underbrace{d(t)}_{\text{dissipated}} \quad (1)$$

where  $x \in \mathcal{R}^n$  is the state vector,  $H(x)$  is the *total energy* function, and  $d(t)$  is a nonnegative function that captures the dissipation effects (e.g., due to resistances and frictions). Energy balancing is, of course, a universal property of physical systems;

therefore, our class, which is nothing other than the well-known *passive* systems, captures a very broad range of applications that include nonlinear and time-varying dynamics.

Two important corollaries follow from (1)

- The energy of the uncontrolled system (i.e., with  $u \equiv 0$ ) is nonincreasing, and it will actually decrease in the presence of dissipation. If the energy function is bounded from below, the system will eventually stop at a point of minimum energy. Also, as expected, the rate of convergence of the energy function is increased if we extract energy from the system, for instance, setting  $u = -K_{di}y$ , with  $K_{di} = K_{di}^T > 0$  a so-called damping injection gain.

- Given that

$$-\int_0^t u^T(s)y(s)ds \leq H[x(0)] < \infty \quad (2)$$

the total amount of energy that can be extracted from a passive system is bounded.

The point where the open-loop energy is minimal (which typically coincides with the zero state) is usually not the one of practical interest, and control is introduced to operate the system around some nonzero equilibrium point, say  $x_*$ . In the standard formulation of PBC, we label the port variables as inputs and outputs (say  $u$  and  $y$ , respectively) and pose the stabilization problem in a classical way [6].

- Select a control action  $u = \beta(x) + v$  so that the closed-loop dynamics satisfies the new energy balancing equation

$$H_d[x(t)] - H_d[x(0)] = \int_0^t v^T(s)z(s)ds - d_d(t)$$

where  $H_d(x)$ , the desired total energy function, has a strict minimum at  $x_*$ ,  $z$  (which may be equal to  $y$ ) is the new passive output, and we have replaced the natural dissipation term by some function  $d_d(t) \geq 0$  to increase the convergence rate.

Later, we will show that this classical distinction between inputs and outputs is restrictive, and the “control-as-interconnection” perspective of Willems is needed to cover a wider range of applications.

### 3 Stabilization via Energy Balancing

There is a class of systems, which interestingly enough includes *mechanical* systems, for which the solution to the problem posed above is very simple, and it reduces to being able to find a function  $\beta(x)$

such that the energy supplied by the controller can be expressed as a function of the state. Indeed, from (1) we see that if we can find a function  $\beta(x)$  such that

$$-\int_0^t \beta^T[x(s)]y(s)ds = H_a[x(t)] + \kappa \quad (3)$$

for some function  $H_a(x)$ , then the control  $u = \beta(x) + v$  will ensure that the map  $v \mapsto y$  is passive with *new energy function*

$$H_d(x) \triangleq H(x) + H_a(x). \quad (4)$$

If, furthermore,  $H_d(x)$  has a minimum at the desired equilibrium  $x_*$ , then it will be stable. Notice that the closed-loop energy is equal to the difference between the stored and the supplied energies. Therefore, we refer to this particular class of PBCs as *energy balancing* PBCs.

Energy-balancing stabilization can, in principle, be applied to general  $(f, g, h)$  nonlinear passive systems of the form

$$\Sigma : \begin{cases} \dot{x} &= f(x) + g(x)u \\ y &= h(x). \end{cases} \quad (5)$$

From the celebrated nonlinear version of the Kalman–Yakubovich–Popov lemma [7], we know that for this class of systems, passivity is *equivalent* to the existence of a nonnegative scalar function  $H(x)$  such that

$$\begin{aligned} \left( \frac{\partial H}{\partial x}(x) \right)^T f(x) &\leq 0 \\ h(x) &= g^T(x) \frac{\partial H}{\partial x}(x). \end{aligned}$$

We have the following simple proposition.

**Proposition 1.** Consider the passive system (5) with storage function  $H(x)$  and an admissible equilibrium  $x_*$ . If we can find vector function  $\beta(x)$  such that the partial differential equation

$$\left( \frac{\partial H_a}{\partial x}(x) \right)^T [f(x) + g(x)\beta(x)] = -h^T(x)\beta(x) \quad (6)$$

can be solved for  $H_a(x)$ , and the function  $H_d(x)$  defined as (4) has a minimum at  $x_*$ , then  $u = \beta(x) + v$  is an energy balancing PBC. Consequently, setting  $v \equiv 0$ , we have that  $x_*$  is a stable equilibrium with Lyapunov function the difference between the stored and the supplied energies.  $\square\square\square$

The proof follows immediately, noting that the left-hand side of (6) equals  $\dot{H}_a$  while the right-hand side is  $-y^T u$ , and then integrating from 0 to  $t$ .

#### 4 Dissipation Obstacle

To investigate the conditions under which the PDE (6) is solvable we note that a necessary condition for the *global* solvability of the PDE (6) is that  $h^\top(x)\beta(x)$  vanishes at all the zeros of  $f(x) + g(x)\beta(x)$ . Now  $f(x) + g(x)\beta(x)$  is obviously zero at the equilibrium  $x_*$ , hence the right-hand side  $-y^\top u$ , which is the *power* extracted from the controller, should also be zero at the equilibrium. This means that energy balancing PBC is applicable only if the energy dissipated by the system is bounded, and consequently if it can be stabilized *extracting a finite amount of energy* from the controller. This is indeed the case in regulation of mechanical systems where the extracted power is the product of force and velocity and we want to drive the velocity to zero. Unfortunately, it is no longer the case for most electrical or electromechanical systems where power involves the product of voltages and currents and the latter may be nonzero for nonzero equilibria.

Let us illustrate this point with simple linear time-invariant RLC circuits. First, we prove that the series RLC circuit is stabilizable with an energy balancing PBC. Then we move the resistance to a parallel connection and show that, since for this circuit the power at any nonzero equilibrium is nonzero, energy balancing stabilization is no longer possible.

##### Finite Dissipation Example

Consider a series RLC circuit, where the port power variables are the input voltage and the current. The “natural” state variables for this circuit are the charge in the capacitor and the flux in the inductance  $x \triangleq [q_C, \phi_L]^\top$ , and the total energy function is

$$H(x) = \frac{1}{2C}x_1^2 + \frac{1}{2L}x_2^2. \quad (7)$$

The dynamic equations are given by

$$\Sigma : \begin{cases} \dot{x}_1 &= \frac{1}{L}x_2 \\ \dot{x}_2 &= -\frac{1}{C}x_1 - \frac{R}{L}x_2 + u \\ y &= \frac{1}{L}x_2 \end{cases} \quad (8)$$

The circuit clearly satisfies (1) with  $d(t) = R \int_0^t [\frac{1}{L}x_2(s)]^2 ds$  (i.e., the energy dissipated in the resistor).

We are given an equilibrium  $x_*$  that we want to stabilize. It is clear from (8) that the admissible equilibria are of the form  $x_* = [x_{1*}, 0]^\top$ . It is important to note that the extracted power at any admissible equilibrium is zero.

To design our energy balancing PBC, we look for a solution of the PDE (6), which in this case takes

the form

$$\frac{x_2}{L} \frac{\partial H_a}{\partial x_1} - \left[ \frac{x_1}{C} + \frac{Rx_2}{L} - \beta(x) \right] \frac{\partial H_a}{\partial x_2} = -\frac{x_2}{L} \beta(x).$$

Notice that the energy function  $H(x)$  already “has a minimum” at  $x_2 = 0$ ; thus we only have to “shape” the  $x_1$  component, so we look for a function of the form  $H_a = H_a(x_1)$ . In this case, the PDE reduces to

$$\beta(x_1) = -\frac{\partial H_a}{\partial x_1}(x_1),$$

which, for any given  $H_a(x_1)$ , defines the control law as  $u = \beta(x_1)$ . To shape the energy  $H_a(x)$ , we add a quadratic term and complete the squares (in the increments  $x - x_*$ ) by proposing

$$H_a(x_1) = \frac{1}{2C_a}x_1^2 - \left( \frac{1}{C} + \frac{1}{C_a} \right) x_{1*}x_1 + \kappa.$$

[The particular notation for the gain  $\frac{1}{C_a}$  will be clarified in the next section.] Replacing in (4), yields

$$H_d(x) = \frac{1}{2} \left( \frac{1}{C} + \frac{1}{C_a} \right) (x_1 - x_{1*})^2 + \frac{1}{2L}x_2^2 + \kappa, \quad (9)$$

which has a minimum at  $x_*$  for all gains  $C_a > -C$ . Summarizing, the control law

$$u = -\frac{x_1}{C_a} + \left( \frac{1}{C} + \frac{1}{C_a} \right) x_{1*} \quad (10)$$

with  $C_a > -C$  is an energy balancing PBC that stabilizes  $x_*$  with a Lyapunov function equal to the difference between the stored and the supplied energy. Finally, it is easy to verify that the energy supplied by the controller is finite.

##### Infinite Dissipation Example

Even though in the previous example we could find a very simple energy balancing solution to our stabilization problem, it is easy to find systems that are not stabilizable with energy balancing PBCs. For instance, consider a parallel RLC circuit. With the same definitions as before, the dynamic equations are now

$$\Sigma : \begin{cases} \dot{x}_1 &= -\frac{1}{RC}x_1 + \frac{1}{L}x_2 \\ \dot{x}_2 &= -\frac{1}{C}x_1 + u \\ y &= \frac{1}{L}x_2 \end{cases} \quad (11)$$

Notice that only the dissipation structure has changed, but the admissible equilibria are now of the form  $x_* = [Cu_*, \frac{L}{R}u_*]^\top$  for any  $u_*$ . The problem is that the power at any equilibrium except the trivial one is nonzero, and consequently any stabilizing controller will yield  $\lim_{t \rightarrow \infty} |\int_0^t u(s)y(s)ds| = \infty$  (we will eventually run out of battery!).

## 5 Overcoming the Dissipation Obstacle

To extend PBC to systems with *infinite dissipation*, we introduce three key modifications. First, since these systems cannot be stabilized by extracting a finite amount of energy from the controller, we consider the latter to be an (infinite energy) *source*; that is, a scalar system

$$\Sigma_c : \begin{cases} \dot{\zeta} &= u_c \\ y_c &= \frac{\partial H_c}{\partial \zeta}(\zeta) \end{cases} \quad (12)$$

with energy function

$$H_c(\zeta) = -\zeta. \quad (13)$$

Second, we incorporate more structure into the system dynamics, in particular, making explicit the damping terms and the dependence on the energy function. Toward this end, we consider port-controlled Hamiltonian models that encompass a very large class of physical nonlinear systems. These models are of the form

$$\Sigma : \begin{cases} \dot{x} &= [J(x) - \mathcal{R}(x)] \frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^\top(x) \frac{\partial H}{\partial x}(x) \end{cases} \quad (14)$$

where  $H(x)$  is the energy function,  $J(x) = -J^\top(x)$  captures the interconnection structure, and  $\mathcal{R}(x) = \mathcal{R}^\top(x) \geq 0$  is the dissipation matrix. Clearly these systems satisfy the energy balancing equation (1).

Third, the classical unitary feedback interconnection (through the power port variables) imposes some very strict constraints on the plant and controller structures as shown in [5]. To provide more design flexibility, we propose to incorporate state information, which is done by coupling the source system with the plant via a *state-modulated* interconnection of the form

$$\begin{bmatrix} u(s) \\ u_c(s) \end{bmatrix} = \begin{bmatrix} 0 & -\beta(x) \\ \beta(x) & 0 \end{bmatrix} \begin{bmatrix} y(s) \\ y_c(s) \end{bmatrix}. \quad (15)$$

This interconnection is clearly power preserving. The overall interconnected system (12), (13), (14), (15) can be written as

$$\begin{bmatrix} \dot{x} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} J(x) - \mathcal{R}(x) & -g(x)\beta(x) \\ \beta^\top(x)g^\top(x) & 0 \end{bmatrix} \begin{bmatrix} \frac{\partial H}{\partial x}(x) \\ \frac{\partial H_c}{\partial \zeta}(\zeta) \end{bmatrix} \quad (16)$$

which is still a port-controlled Hamiltonian system with total energy  $H(x) + H_c(\zeta)$ . It is important to note that the  $x$  dynamics above describes the behavior of the system (14) with a static state feedback  $u = \beta(x)$ ; hence our choice of the symbol  $\beta$  for the state-modulation function.

Energy of the  $x$  subsystem can be shaped and the port-controlled Hamiltonian structure preserved as follows. If (for the given  $J(x), \mathcal{R}(x)$  and  $g(x)$ ) we can solve the PDE

$$[J(x) - \mathcal{R}(x)] \frac{\partial H_a}{\partial x}(x) = g(x)\beta(x) \quad (17)$$

for some  $\beta(x)$ , then the plant dynamics will be given by

$$\dot{x} = [J(x) - \mathcal{R}(x)] \frac{\partial H_d}{\partial x}(x) \quad (18)$$

with energy function  $H_d(x) = H(x) + H_a(x)$ . If we can furthermore ensure that  $H_d(x)$  has a minimum at the desired equilibrium, then the static state feedback control  $u = \beta(x)$  will stabilize this point. Notice that there is no “finite dissipation” constraint for the solvability of (17); hence the new PBC design is, in principle, applicable to systems with infinite dissipation.

### Parallel RLC circuit example

Before presenting the main result of this section, which is a systematic procedure for PBC of port-controlled Hamiltonian systems, let us illustrate the new energy shaping method with the parallel RLC circuit example. The dynamics of this circuit (11) can be written in port-controlled Hamiltonian form (14) with energy function (7) and the matrices

$$J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \mathcal{R} = \begin{bmatrix} 1/R & 0 \\ 0 & 0 \end{bmatrix}, \quad g = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

The PDE (17) becomes

$$\begin{aligned} -\frac{1}{R} \frac{\partial H_a}{\partial x_1}(x) + \frac{\partial H_a}{\partial x_2}(x) &= 0 \\ -\frac{\partial H_a}{\partial x_1}(x) &= \beta(x). \end{aligned}$$

The first equation can be trivially solved as

$$H_a(x) = \Phi(Rx_1 + x_2)$$

where  $\Phi(\cdot) : \mathcal{R} \rightarrow \mathcal{R}$  is an arbitrary differentiable function, whereas the second equation defines the control law. We now need to choose the function  $\Phi$  so that  $H_d(x)$  has a minimum at the desired equilibrium point  $x_* = (Cu_*, \frac{L}{R}u_*)$ . For simplicity, we choose it to be a quadratic function

$$\Phi = \frac{K_p}{2} [(Rx_1 + x_2) - (Rx_{1*} + x_{2*})]^2 - Ru_*(Rx_1 + x_2)$$

which, as can be easily verified, ensures the desired energy shaping for all

$$K_p > \frac{-1}{(L + CR^2)}. \quad (19)$$

The assigned energy function, as expected, is quadratic in the increments

$$H_d(x) = (x - x_*)^\top \begin{bmatrix} \frac{1}{C} + R^2 K_p & RK_p \\ RK_p & \frac{1}{L} + K_p \end{bmatrix} (x - x_*).$$

Clearly, (19) is the necessary and sufficient condition for  $x_*$  to be a unique global minimum of this function. The resulting control law is a simple linear state feedback

$$u = -K_p[R(x_1 - x_{1*}) + x_2 - x_{2*}] + u_*.$$

## 6 Assigning Interconnection and Damping Structures

In the previous subsections, we have shown that the success of our PBC design essentially hinges on our ability to solve the PDE (17). It is well known that solving PDEs is not easy. It is our contention that, for the particular PDE that we have to solve here, it is possible to incorporate prior knowledge about the system to simplify the task. More specifically, for port-controlled Hamiltonian models, besides the control law, we have the additional degrees of freedom of selecting the interconnection and damping structures of the closed-loop. Indeed, our energy shaping objective is not modified if, instead of (18), we aim at the closed-loop dynamics

$$\dot{x} = [J_d(x) - \mathcal{R}_d(x)] \frac{\partial H_d}{\partial x}(x) \quad (20)$$

for some *new* interconnection  $J_d(x) = -J_d^\top(x)$  and damping  $\mathcal{R}_d(x) = \mathcal{R}_d^\top(x) \geq 0$  matrices. For this so-called interconnection and damping assignment PBC the PDE (17) becomes

$$[J(x) + J_a(x) - \mathcal{R}(x) - \mathcal{R}_a(x)] \frac{\partial H_a}{\partial x}(x) = \quad (21)$$

$$- [J_a(x) - \mathcal{R}_a(x)] \frac{\partial H}{\partial x}(x) + g(x)\beta(x) \quad (22)$$

where

$$J_a(x) \triangleq J_d(x) - J(x), \quad \mathcal{R}_a(x) \triangleq \mathcal{R}_d(x) - \mathcal{R}(x)$$

are new design parameters that add more degrees of freedom to the solution of the PDE.

The proposition below (established in [5]) follows immediately from the derivations above. It is presented in a form that is particularly suitable for symbolic computations. We refer the interested reader to [5] for additional comments and discussions.

**Proposition 2.** Given  $J(x), \mathcal{R}(x), H(x), g(x)$ , and the desired equilibrium to be stabilized  $x_*$ , assume

we can find functions  $\beta(x), \mathcal{R}_a(x), J_a(x)$  such that

$$\begin{aligned} J(x) + J_a(x) &= -[J(x) + J_a(x)]^\top \\ \mathcal{R}(x) + \mathcal{R}_a(x) &= [\mathcal{R}(x) + \mathcal{R}_a(x)]^\top \geq 0 \end{aligned}$$

and a vector function  $K(x)$  satisfying

$$\begin{aligned} [J(x) + J_a(x) - (\mathcal{R}(x) + \mathcal{R}_a(x))]K(x) &= \\ - [J_a(x) - \mathcal{R}_a(x)] \frac{\partial H}{\partial x}(x) + g(x)\beta(x) \end{aligned} \quad (23)$$

and such that the following conditions occurs

(i) (*Integrability*)  $K(x)$  is the gradient of a scalar function; that is,

$$\frac{\partial K}{\partial x}(x) = \left[ \frac{\partial K}{\partial x}(x) \right]^\top \quad (24)$$

(ii) (*Equilibrium assignment*)  $K(x)$ , at  $x_*$ , verifies

$$K(x_*) = -\frac{\partial H}{\partial x}(x_*).$$

(iii) (*Lyapunov stability*) The Jacobian of  $K(x)$ , at  $x_*$ , satisfies the bound

$$\frac{\partial K}{\partial x}(x_*) > -\frac{\partial^2 H}{\partial x^2}(x_*).$$

Under these conditions, the closed-loop system  $u = \beta(x)$  will be a port-controlled Hamiltonian system with dissipation of the form (20), where  $H_d(x) = H(x) + H_a(x)$  and

$$\frac{\partial H_a}{\partial x}(x) = K(x). \quad (25)$$

Furthermore,  $x_*$  will be a (locally) *stable* equilibrium of the closed loop. It will be *asymptotically* stable if, in addition, the largest invariant set under the closed-loop dynamics contained in

$$\left\{ x \in \mathcal{R}^n \cap B \mid \left[ \frac{\partial H_d}{\partial x}(x) \right]^\top \mathcal{R}_d(x) \frac{\partial H_d}{\partial x}(x) = 0 \right\}$$

equals  $\{x_*\}$ .

□□□

## 7 Concluding Remarks

We have given a tutorial presentation of a control design approach for physical systems based on energy considerations that has been developed by the authors of the present article, as well as by some other researchers cited in the references, in the last few years. The main premise of this approach is

that the fundamental concept of energy is lost in the signal processing perspective of most modern control techniques, hence we present an alternative viewpoint which focuses on interconnection. The choice of a suitable description of the system is essential for this research, thus we have adopted port-controlled Hamiltonian models which provide a classification of the variables and the equations into those associated to phenomenological properties and those defining the interconnection structure related with the exchanges of energy.

There are many possible extensions and refinements to the theory we have presented in this article. Some of these topics, and the lines of research we are pursuing to address them, may be found in [5]. Central among the various open issues that need to be clarified one finds, of course, the solvability of the PDE (23). Although we have shown that the added degrees of freedom ( $J_a(x), \mathcal{R}_a(x)$ ) can help us in its solution, it would be desirable to have a better understanding of their effect, that would lead to a more systematic procedure in their design. For general port-controlled Hamiltonian systems this is, we believe, a far-reaching problem. Hence, we might want to study it first for specific classes of physically-motivated systems.

Solving new problems is, of course, the final test for the usefulness of a new theory. The existing applications of interconnection and damping assignment PBC include mass-balance systems [8], electrical motors [9], power systems [10], magnetic levitation systems [11], underactuated mechanical systems [4], and power converters [12]. Our list of references witnesses to the breadth of application of our approach, hence we tend to believe that this aspect has been amply covered by our work.

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